

## Ch Centre of gravity

Defn: The centre of gravity of a body or a system of particles rigidly connected together is that pt. through which the line of action of the wt. of the body always passes, in whatever position the body is placed. It is briefly denoted by CG.

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Th

Ex- Let the

Find the CG. of a system of particles lying in a vertical plane.



$A_1(x_1, y_1), A_2(x_2, y_2)$   
 $\dots$   
 $A_3(x_3, y_3)$   
 $\dots$   
 $G(\bar{x}, \bar{y})$

Soln: Let the particles of wt.  $w_1, w_2, w_3, \dots$  be placed at pts.  $A_1, A_2, A_3, \dots$

Let us take lines  $ox, oy$  at st. angles to each other as the axes of co-ordinates and let  $(x_1, y_1), (x_2, y_2), (x_3, y_3), \dots$  are the co-ordinates of the pts  $A_1, A_2, A_3, \dots$  referred to the axes  $ox$  and  $oy$  and let  $(\bar{x}, \bar{y})$  be the

co-ordinates of the C.G. of the system which is the RTG. (say).

Now the position of G relative to the plane of the particles, does not depend on the position of the plane. Therefore for finding  $\bar{x}$ , we assume that the plane is vertical and is so placed that x-axis is horizontal. Consequently, the weights  $w_1, w_2, w_3, \dots$  etc. all like parallel forces, parallel to y-axis and their resultant  $R = w_1 + w_2 + w_3 + \dots$  is also parallel force parallel to y-axis acting through the RTG ( $\bar{x}, \bar{y}$ ).

Hence taking moment about O, we get

$$\begin{aligned}
 R \cdot \bar{x} &= w_1 x_1 + w_2 x_2 + w_3 x_3 + \dots \\
 \Rightarrow \bar{x} &= \frac{w_1 x_1 + w_2 x_2 + \dots}{w_1 + w_2 + \dots} \\
 &= \frac{\sum w_i x_i}{\sum w_i} \quad \text{--- (1)}
 \end{aligned}$$

For finding  $\bar{y}$ , we assume the plane

is vertical and is so placed that  $y$  is horizontal, so that the weights and their resultant are like parallel forces, parallel to  $x$ -axis. Taking moments about  $O$ , we get in this case,

$$\begin{aligned} \bar{y} &= \frac{w_1 y_1 + w_2 y_2 + \dots}{w_1 + w_2 + \dots} \\ &= \frac{\sum w_i y_i}{\sum w_i} . \quad (2) \end{aligned}$$

Eqs (1) & (2) give the co-ordinates of G.

Determination of C.G. by Integral Calculus:

$$\bar{x} = \frac{\int x dw}{\int dw} \quad \& \quad \bar{y} = \frac{\int y dw}{\int dw} .$$

If  $m$  be the mass of the elementary portion whose weight is  $dw$ , then  $dw = dm g$  and the above formulae takes the form

$$\bar{x} = \frac{\int x dm}{\int dm}, \quad \bar{y} = \frac{\int y dm}{\int dm}$$

Here  $dm$  is the mass of the elementary portion of the given matter and  $(\bar{x}, \bar{y})$  is the C.G. of the element of mass  $dm$ .

$O \rightarrow 86, 88, 90, 93, 98.$

$M \rightarrow 95, 99$

Ex- A uniform wire is bent into an arc of a circle of radius  $a$ , the arc subtends an angle  $2\alpha$  at the centre F and the C.G.

Sol: At  $G(\bar{x}, \bar{y})$  be the C.G. of the uniform

wire bent into  
the form of an  
arc of a circle.

Let  $O$  be the centre  
of the circular arc.

Let  $\angle AOB = 2\alpha$ . Let  $M$

be the middle pt of the arc  $AB$ .  $OM$  is joined  
and produced to  $X$ . Taking  $O$  as the origin  
and  $OX$  as the  $x$ -axis, we have by sym-  
metry,  $\bar{Y} = 0$ . Let  $\angle XOP = \theta$

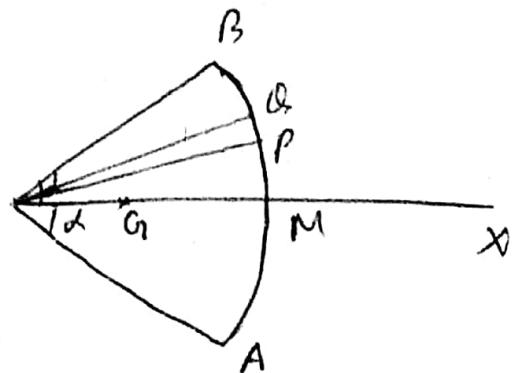
Let  $PQ$  be a very small arc. We  
have the length of the arc  $MP$ ,  $s = a\theta$ .

$$\therefore ds = a d\theta$$

$$\therefore \bar{x} = \frac{\int x ds}{\int ds} = \frac{\int a \cos \theta \cdot a d\theta}{\int a d\theta}$$

$$= a \cdot \frac{[\sin \theta]_{-\alpha}^{\alpha}}{[\theta]_{-\alpha}^{\alpha}} = a \frac{\sin \alpha}{\alpha}$$

$\therefore$  The C.G. is at  $(a \frac{\sin \alpha}{\alpha}, 0)$ . //



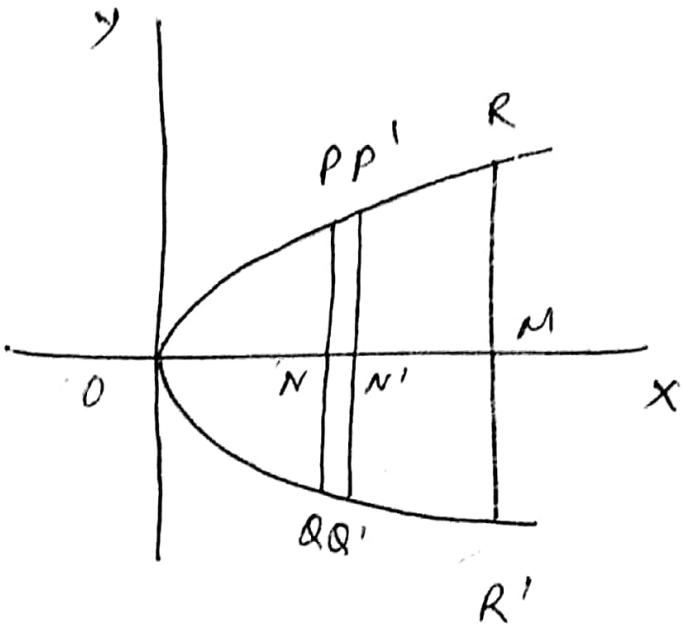
G → 87, 89, 91, 03, 205, 07

M → 95, 2002, 205

Ex. 4 Find the C.G. of a uniform lamina bounded by a parabola and a double ordinate. (by  $y^2 = 4ax$ , the x-axis and its radius vector)   
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Soln Let the lamina be bounded by a parabola  $y^2 = 4ax$  and a double ordinate  $RMR'$  given by  $x = x_1$ .

By symmetry, the centroid lies on the axis  $y=0$ .



Dividing the lamina by elementary strips by lines parallel to the y-axis and considering the strip  $PQQP'$  where the co-ordinates of  $P$  are  $(x, y)$ . we have the area of the strip corresponding to  $P = 2y \cdot dx$ . The limits of  $x$  to cover the area considered are clearly from 0 to  $M$  i.e. 0 to  $x_1$ . let  $P$  be density of the lamina.

$\therefore$  The required abscissa of C.G.

$$\bar{x} = \frac{\int_0^{x_1} x \cdot 2y dx}{\int_0^{x_1} 2y dx} = \frac{\int_0^{x_1} x \cdot 2\sqrt{a^2 - x^2} dx}{\int_0^{x_1} 2\sqrt{a^2 - x^2} dx}$$

$$= \frac{\int_0^{x_1} x^{3/2} dx}{\int_0^{x_1} x^{1/2} dx} = \frac{\frac{2}{5} x_1^{5/2}}{\frac{2}{3} x_1^{3/2}} = \frac{3}{5} x_1$$

Thus the C.G. divides the length OM in

The ratio 3:5.

G  $\rightarrow$  94, 96

Cyr. :- The C.G. of the lamina bounded

by a parabola and its latus rectum

is given by

$$\bar{x} = \frac{\int_0^a x^{3/2} dx}{\int_0^a x^{1/2} dx} = \frac{3}{5} a. \text{ & } \bar{y} = 0.$$

No 2 old  
This C.G. of quadrant of ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . (6)

Q8- To find the C.G. of a sectorial area bounded by curve  $r = f(\theta)$ , radii vectors  $\theta = \alpha$  and  $\theta = \beta$ .

Soln. Let us take two neighbouring pt.  $P(r, \theta)$  and  $Q(r + \delta r, \theta + \delta\theta)$  on the curve. Let us join  $OP$  and  $OQ$ . The sector  $POQ$  is the elementary area under consideration. The area of the sector  $POQ = \frac{1}{2}r^2\delta\theta$ .

Also if  $G'$  be its C.G., Then  $OG' = \frac{2}{3}OP$  in the limit (as sector  $POQ$  behaves like a triangle).

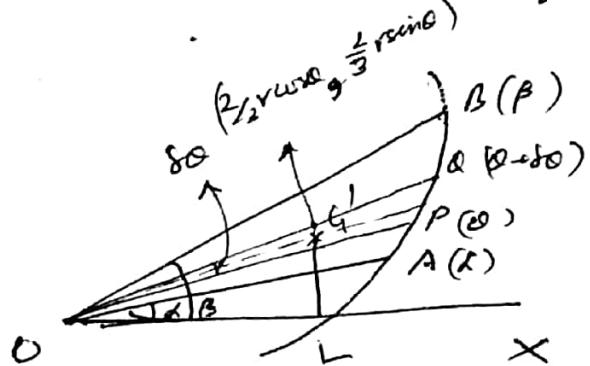
$\therefore$  Its C.G. is the pt. of intersection of medians i.e.  $OG' = \frac{2}{3}r$ .

$\therefore$  Co-ordinates of  $G'$  are  $(OL, G'L)$ , i.e.,  $(\frac{2}{3}r \cos\theta, \frac{2}{3}r \sin\theta)$ .

Let  $\rho$  be the mass per unit area.

Then  $\delta m$  = mass of the elementary area  $= \rho \cdot \frac{1}{2}r^2\delta\theta$ .

$\therefore$  If  $(\bar{x}, \bar{y})$  be the required C.G. of sector  $AOB$  and  $\rho = \text{constant}$ , then



$$\begin{aligned}
 \bar{x} &= \frac{\int x dm}{\int dm} \\
 &= \frac{\int_{\theta=\alpha}^{\beta} \frac{2}{3} r \cos \theta \cdot p \cdot \frac{1}{2} r^2 d\theta}{\int_{\theta=\alpha}^{\beta} p \cdot \frac{1}{2} r^2 d\theta} \\
 &= \frac{2}{3} \frac{\int_{\theta=\alpha}^{\beta} r^3 \cos \theta d\theta}{\int_{\theta=\alpha}^{\beta} r^2 d\theta} \\
 &\quad \times \bar{y} = \frac{\int x dm}{\int dm} = \frac{\int_{\theta=\alpha}^{\beta} \frac{2}{3} r \sin \theta p \cdot \frac{1}{2} r^2 d\theta}{\int_{\theta=\alpha}^{\beta} p \cdot \frac{1}{2} r^2 d\theta} \\
 &= \frac{2}{3} \cdot \frac{\int_{\theta=\alpha}^{\beta} r^3 \sin \theta d\theta}{\int_{\theta=\alpha}^{\beta} r^2 d\theta}.
 \end{aligned}$$